2.9 Lower bound calculation

In the lower bound calculation, admissible stress field is needed to be found.

Admissible stress field:

1. **Equilibrium of forces and stresses should be satisfied both in soil and boundary**
2. **Failure condition is not violated everywhere in soil.**

Equilibrium conditions

Two types of equilibrium:

1) continuous change of stress

2) discontinuous stress

1) continuous change of stress

for undrained conditions

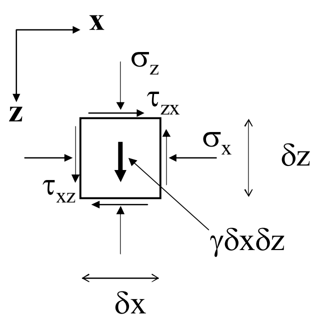
$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma \quad (4.48)$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0 \quad (4.49)$$

for drained conditions

$$\frac{\partial \sigma'_z}{\partial z} + \frac{\partial \tau'_{xz}}{\partial x} = \gamma - \frac{\partial u}{\partial z} \quad (4.50)$$

$$\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau'_{zx}}{\partial z} = -\frac{\partial u}{\partial x} \quad (4.51)$$



Discontinuous stress state

2) discontinuous stress
（不連続応力）

<u>for undrained conditions</u>

$$\sigma_{na} = \sigma_{nb}$$

$$\tau_{na} = \tau_{nb} = -\tau_{ta} = -\tau_{tb} \quad (4.52)$$

σ_{ta} and σ_{tb} need not be equal for equilibrium.

<u>for drained conditions</u>

$$\sigma'_{na} = \sigma'_{nb} \quad \tau'_{na} = \tau'_{nb} \quad (4.53)$$

u_a and u_b at both sides of discontinuity are equal.

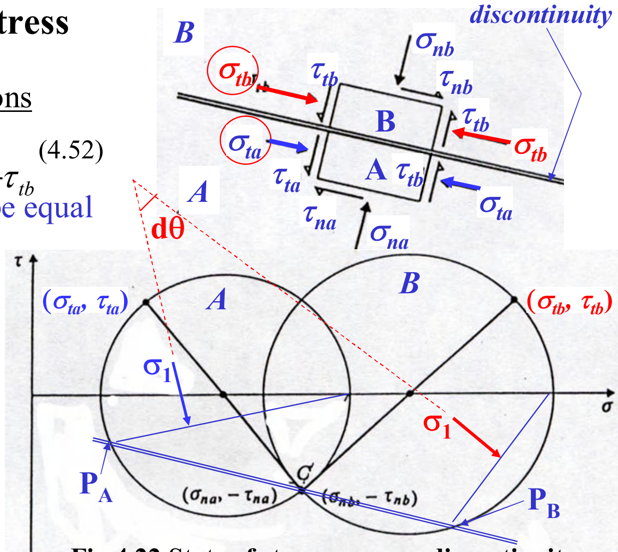


Fig.4.22 State of stress across a discontinuity

Directions of major principal stress across a discontinuity

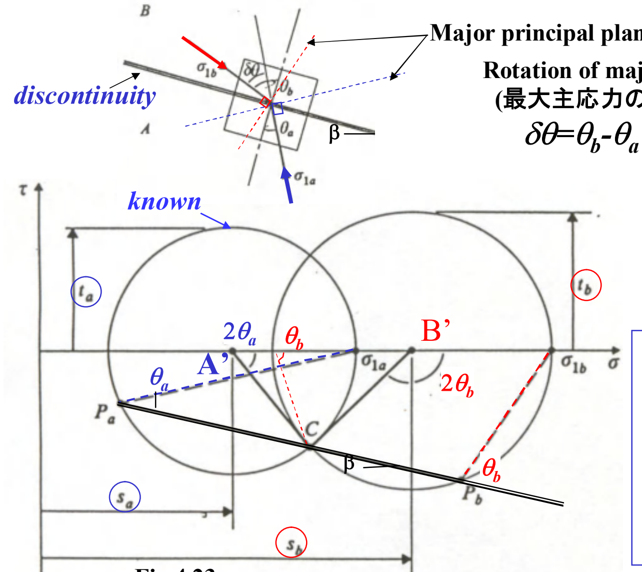


Fig.4.23

Major principal plane

Rotation of major principal stress:
(最大主応力の回転量)

$\delta\theta = \theta_b - \theta_a$

Mean principal stress
Max. shear stress

$$s = \frac{\sigma_1 + \sigma_3}{2}, \quad t = \frac{\sigma_1 - \sigma_3}{2}$$

Known principal stress vector of A (s_a, t_a, β, θ_a)
+
Rotation of σ_1($\delta\theta = \theta_b - \theta_a$)
⇩
Stress of B
(s_b, t_b)

Change of stress across a discontinuity for undrained loading

Failure criteria

$\tau = t = c_u$

Size of Mohr circle

$$\delta s = 2c_u \sin \delta\theta \quad \textbf{(4.55)}$$

Increment of mean normal stress associated with rotation of principal stress: $\delta\theta$

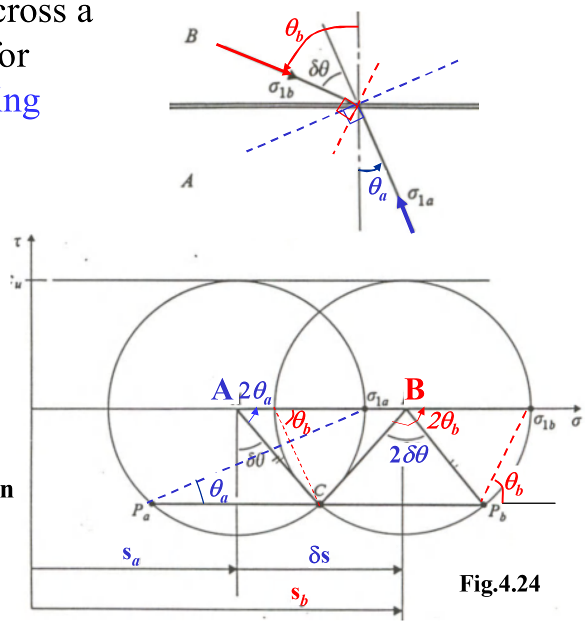


Fig.4.24

Change of stress across a discontinuity for drained loading

Failure criteria

$\tau_f = \sigma'_n \tan\phi'$

$t = s' \sin\phi'$

(4.55)

Mobilized friction angle on discontinuity

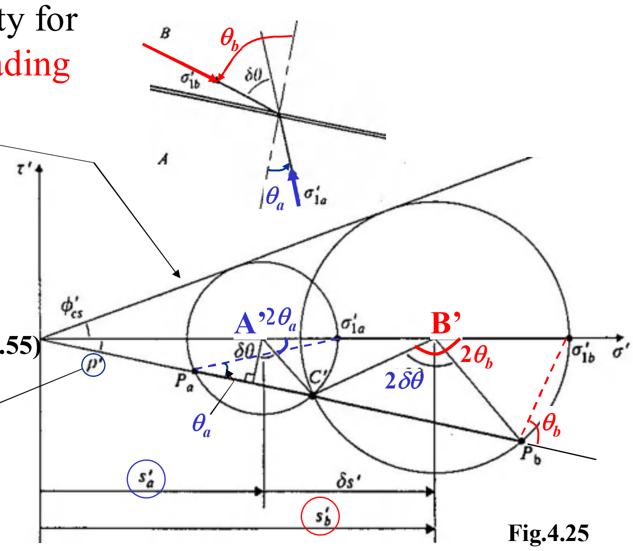


Fig.4.25

Cont.

$$\sin P = \frac{A'D'}{t'_a} \quad (1), \sin \rho' = \frac{A'D'}{s'_a} \quad (2)$$

$$P = 90^\circ - \delta\theta \quad (3)$$

From (1),(2)

$$\sin \rho' = \frac{t'_a}{s'_a} \sin P = \sin \phi' \sin P$$

$$= \cos \delta\theta \sin \phi' \quad (4.60)$$

$$\frac{O'E'}{s'_a} = \sin(P + \rho')$$

$$\frac{O'F'}{s'_b} = \sin(P - \rho')$$

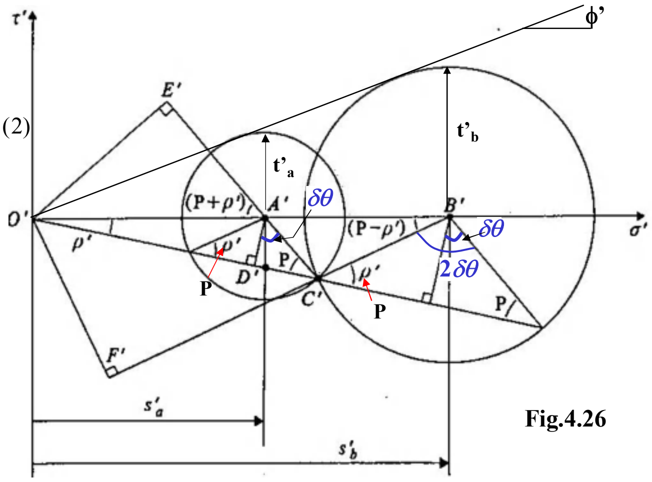


Fig.4.26

Since O'E'=O'F'

$$\frac{s'_b}{s'_a} = \frac{\sin(P + \rho')}{\sin(P - \rho')} = \frac{\cos(\delta\theta - \rho')}{\cos(\delta\theta + \rho')} \quad (4.64)$$

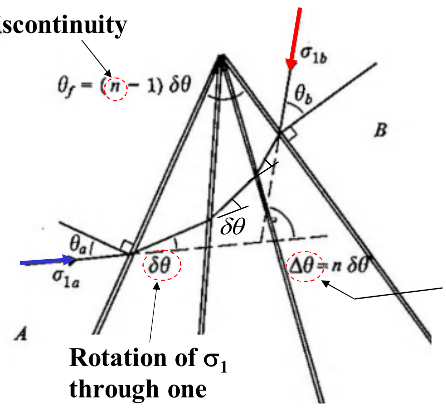
ρ' can be eliminated using (4.60)

δθ => ***not increment but ratio***

Stress Fan

Transient zone between two stress conditions (A&B)

Number of discontinuity



Rotation of σ_1 from A to B

Rotation of σ_1 through one discontinuity

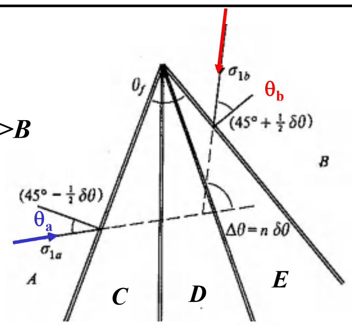
Fig.4.27

State of stress across a fan of discontinuities for undrained loading

A=>C=>D=>E=>B

5 regions

5 Mohr circles



Increment of s (δs) in one discontinuity:

$$\delta s = 2c_u \sin\delta\theta$$

Increment of s (δs) in n discontinuities:

$$\delta s = n(2c_u \sin\delta\theta)$$

$$= n\left\{2c_u \sin\left(\frac{\theta_f}{n-1}\right)\right\}$$

(4.67)

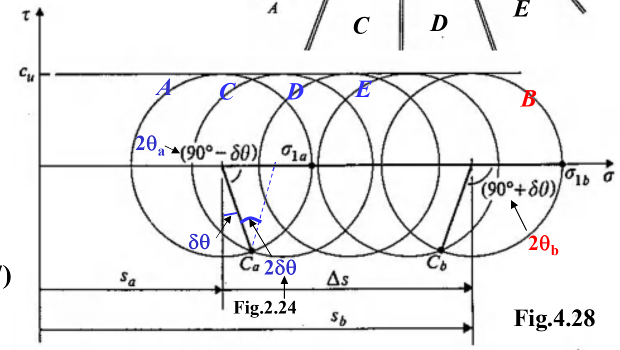


Fig.2.24

Fig.4.28

Stress fan for undrained loading infinite discontinuities

$n \to \infty, \quad \delta\theta \to 0$

$\theta_a = 45^\circ, \theta_b = 45^\circ$

$$\frac{ds}{d\theta} = 2c_u$$

$$\Delta s = s_b - s_a = 2c_u\theta_f$$

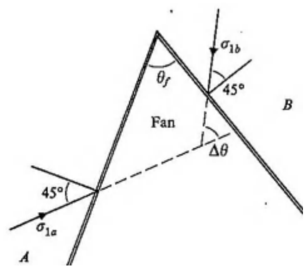
$$\Delta s = 2c_u\Delta\theta \qquad (4.69)$$

Ex) $\Delta\theta = \dfrac{\pi}{2}$

Fan => $\Delta s = \pi c_u$

1 discont. => $\Delta s = 2c_u$

(4.55)



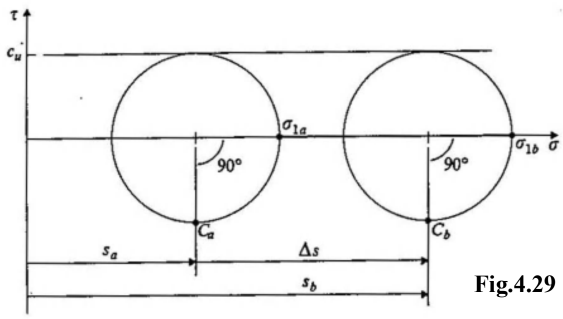


Fig.4.29

State of stress across a fan of discontinuities for drained loading

Stress change due to one discontinuity

$$\frac{s'_b}{s'_a} = \frac{\cos(\delta\theta - \rho')}{\cos(\delta\theta + \rho')} \quad (4.64)$$

$$\frac{\delta s'}{s'} = 2\frac{\sin\delta\theta \sin\rho'}{\cos(\delta\theta + \rho')} \quad (4.71)$$

$$n \to \infty, \quad \delta\theta \to 0$$

$$\theta_a = 45^\circ + \frac{\phi'}{2},$$

$$\theta_b = 45^\circ + \frac{\phi'}{2}$$

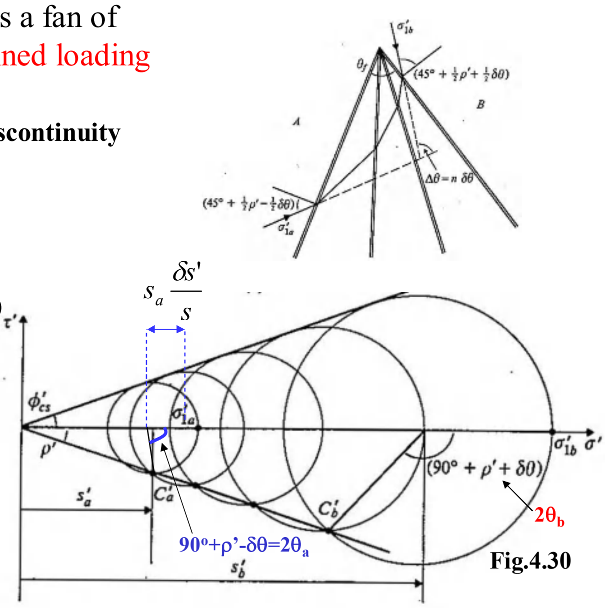


Fig.4.30

Stress fan for undrained loading infinite discontinuities

$$\sin\rho' = \cos\delta\theta \sin\phi' \quad (4.60)$$
$$= \sin\phi'$$

$$\frac{\delta s'}{s'} = 2\frac{\sin\delta\theta \sin\rho'}{\cos(\delta\theta + \rho')} \quad (4.71)$$

$$\frac{ds}{d\theta} = 2s'\tan\phi' \quad (4.72)$$

Sだ

$$\frac{s'_b}{s'_a} = \exp[2\theta_f \tan\phi']$$
$$= \exp[2\Delta\theta \tan\phi'] \quad (4.73)$$

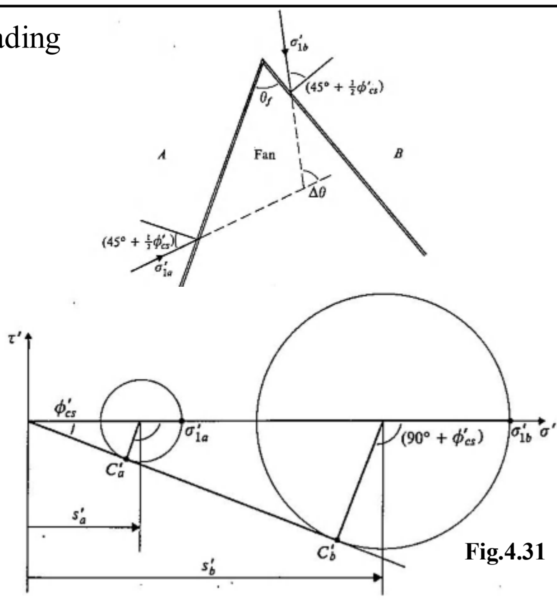


Fig.4.31

Increment of mean normal stress is given by the rotation of principal stress

Change of stress across discontinuity

Change of stress state across a discontinuity is simply related to the rotation δθ of the direction of the major principal stress.

<u>for general case</u>: Fig.4.23, 4.27

<u>for undrained loading</u>:

-single discontinuity (Fig.4.24) $\delta s = \pm 2c_u \sin\delta\theta$ (4.76)

-stress fan (Figs.4.28& 29) $\dfrac{ds}{d\theta} = \pm 2c_u$ (4.78)

<u>for drained loading:</u>

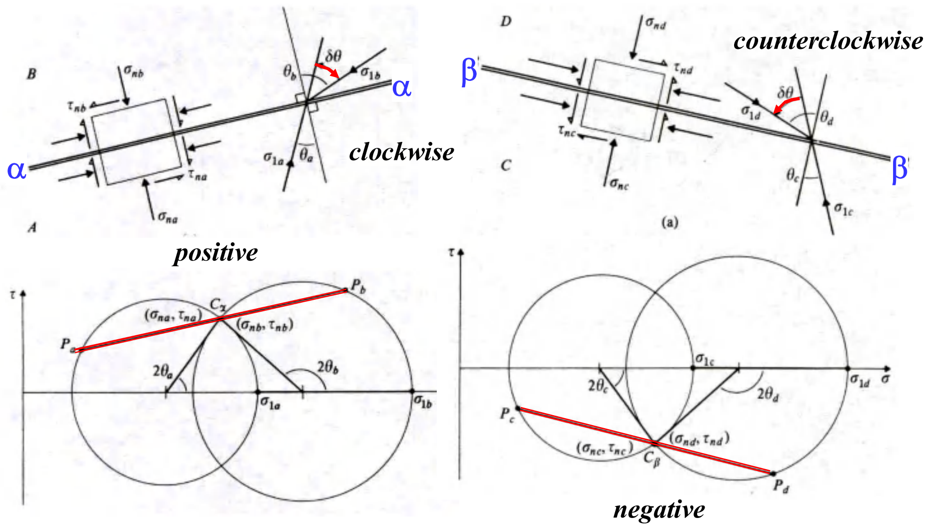
-single discontinuity (Fig.4.25) $\delta s' = \pm 2s' \dfrac{\sin\delta\theta \sin\rho'}{\cos(\delta\theta + \rho')}$ (4.77)

-stress fan (Figs.4.30& 31) $\dfrac{ds'}{d\theta} = \pm 2s' \tan\phi'$ (4.79)

$\pm$ ← **β discontinuity (Fig.33)** ; ← **α discontinuity (Fig.32)**

where $\sin\rho' = \cos\delta\theta \sin\phi'$

α discontinuity & β discontinuity
(s_1 slip lines & s_2 slip line)



Example of upper bound and lower bound calculations

Chapter 5 for undrained stability of soil structures

Chapter 6 for drained stability of soil structures

Vertical cut, retaining wall, shallow foundation are solved using various admissible velocity fields (failure mechanisms) and admissible stress fields.

Home work 4 due date 8th of Nov.

Stress filed of ϕ_u=0 material.

Fig1 and Fig.2 show the permissible stress field of ϕ_u=0 material under footing load with embedment of D.

Draw the Mohr's stress circles for each area like Figure 5.17 and 5.19 in the text book and confirm these are both permissible stress field.

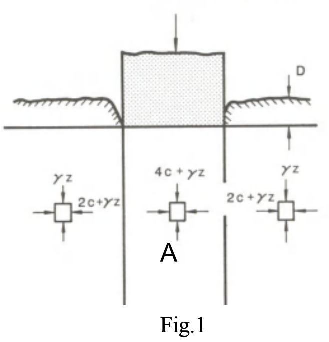


Fig.1

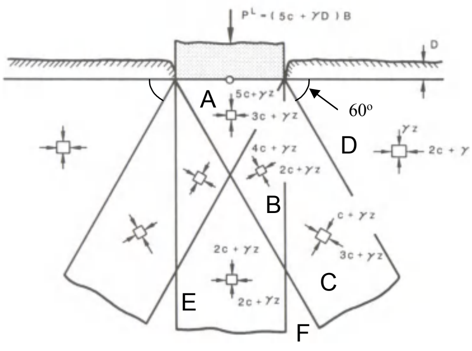
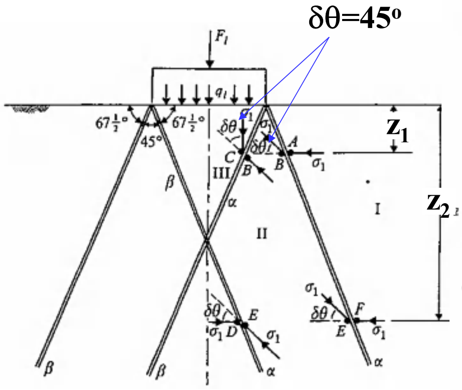
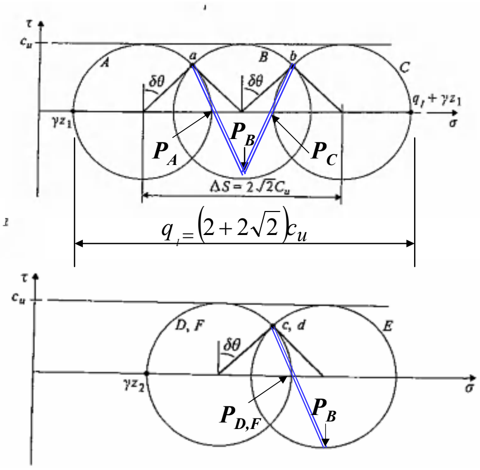


Fig.2

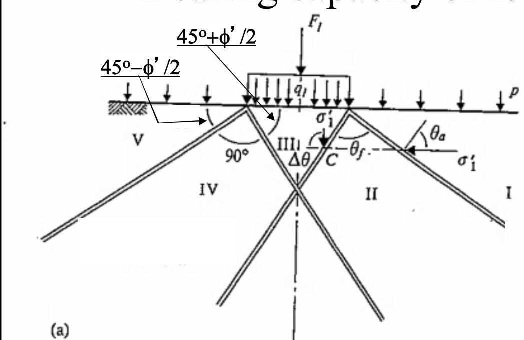
Example of lower bound calculation
Bearing capacity of foundation on clay (c_u)



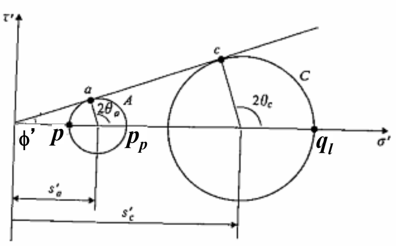


$$q_l = \left(2+2\sqrt{2}\right)c_u$$

Lower bound calculation
Bearing capacity of foundation on sand



weightless: γ=0, surcharge: *p*



$$\frac{s'_c}{s'_a} = \frac{q_l}{p_p} = \exp[2\Delta\theta \tan\phi'] \quad (4.73)$$

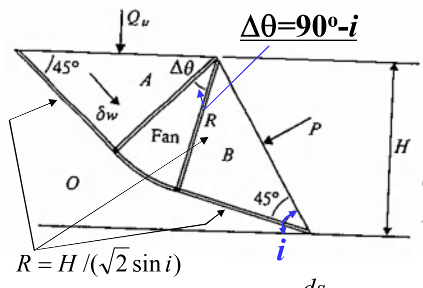
$$=> q_l = p_p \exp[2\Delta\theta \tan\phi']$$

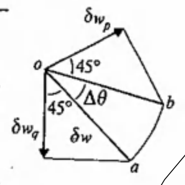
$$(\because p_p = p\tan^2(45^o + \phi'/2), \Delta\theta = \pi/2)$$

$$q_l = p\tan^2(45^o + \phi'/2)\exp[\pi \tan\phi']$$ ⇨ **same as q_u given in (6.76)**

Example of upper bound and lower bound calculations bearing capacity of loaded clay slope

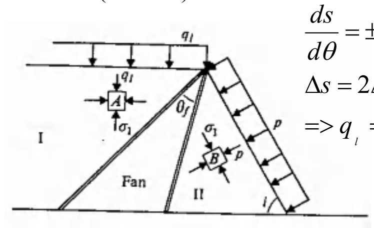
$\Delta\theta$=90º-*i*





p186 => γ=0

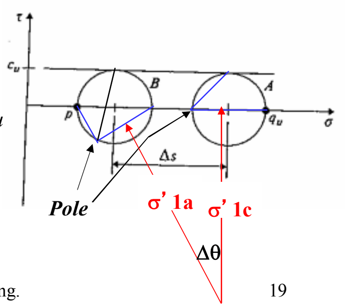
If γ ≠ 0, q_u?, q_l?



$$\frac{ds}{d\theta} = \pm 2c_u \quad \leftarrow \quad (4.78)$$

$$\Delta s = 2\Delta\theta c_u$$

$$\Rightarrow q_l = p + (2 + \pi - 2i)c_u$$



Weightless material: γ=0 **γ ≠ 0**

$$P = pH/\sin i, \qquad Q_u = q_u H/\sin i$$

$$\delta w_p = \delta w_q = \frac{1}{\sqrt{2}}\delta w,$$

\+ **δE done by body force > 0**

$$\Delta E = Q_u \delta w_q - P\delta w_p = \frac{H}{\sqrt{2}\sin i}(q_u - p)\delta w$$

$$\Delta W = \sum c_u \cdot L \cdot \delta w + \sum 2c_u R\Delta\theta\delta w = \frac{\sqrt{2}c_u H\delta w}{\sin i}\left(1 + \frac{1}{2}\pi - i\right)$$

$$\frac{\sqrt{2}c_u H}{\sin i}\delta w \qquad \frac{\sqrt{2}c_u H}{\sin i}\left(\frac{1}{2}\pi - i\right)\delta w$$

$$\Delta E = \Delta W \Rightarrow \quad q_u = c_u(2 + \pi - 2i) + p$$

same as q_u

q_u < q_u for γ=0

Confirm yourself